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The work during the period of the grant included analysis of the stability and instability of mechanical and aerospace systems in the presence of dissipation; work on the geometry, control and stabilization of mechanical systems with nonholonomic constraints; work on the solution of various optimal control problems arising in mechanics; work on the solutions and geometry of partial differential equations describing mechanical and fluid systems; and work on the stabilization of systems modelling anti-corrosion processes.

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Final Technical Report
AFOSR Grant: F49620-93-1-0037
Anthony M. Bloch

My work over the period of the grant includes analysis of the stability of mechanical and fluid systems in the presence of dissipation; work on the geometry, control and stabilization of systems with nonholonomic constraints; work on optimal control; work on the geometry of certain PDE's; and work on the stabilization of systems arising in anti-corrosion processes.

With Krishnaprasad, Marsden and Ratiu, I have been analyzing the stability of mechanical systems with symmetries and, in particular, the effect that dissipative perturbations have on gyroscopically stable equilibria, that is, equilibria which are spectrally stable but have indefinite second variation of the augmented Hamiltonian. In earlier work we considered dissipation in internal or vibrational variables. Recently we have considered dissipation or damping in rotational variables as well. This leads us to a type of dissipation in double bracket form.

To demonstrate the type of dissipative mechanism we have in mind, we now give a simple example of it for the rigid body. Here the underlying space is that of the rotation group, that is, Euclidean three-space $\mathbb{R}^3$ interpreted as the space of body angular velocities Ω equipped with the cross product as the Lie bracket. On this space, we put the standard kinetic energy Lagrangian $L(\Omega) = \frac{1}{2}(I\Omega)\cdot\Omega$ where I is the moment of inertia tensor. The equations for a freely spinning rigid body in terms of the body angular momentum $M = I\Omega$ are $\dot{M} = M \times \Omega$. Now we modify the equations by adding a term cubic in the angular velocity:

$$\dot{M} = M \times \Omega + \alpha M \times (M \times \Omega),$$

where α is a positive constant.

One can check that the addition of the dissipative term has a number of interesting properties. First of all, this dissipation is derivable from an $SO(3)$-invariant force field, but it is not induced by *any* Rayleigh dissipation function in the *literal* sense. However, it is induced by a Rayleigh dissipation function in the following sense: it is a gradient when restricted to each momentum sphere. One can view this as a low dimensional model for a satellite with ring damping say, and one can show that spin about the minor axis is unstable in this case (as well as spin about the intermediate axis as usual). This procedure in fact gives us a good general method for modeling systems where certain conserved quantities such as momentum are respected by the damping.

Another interesting system we have considered is a satellite with momentum wheels. We have derived a number of general stability and instablity results for systems of this type where we can handle both dissipation in the wheels and damping of the overall motion.

One can induce similar dissipation in a perfect fluid:

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla p \tag{1.1}$$

where v is the velocity field, assumed divergence free and parallel to the boundary of the fluid container, and where p is the pressure. With dissipation the equations become:

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = -\nabla p + \alpha \mathbf{P}\left(\left(\pounds_{u(v)} v^\flat\right)^\sharp\right) \tag{1.2}$$

where α is a positive constant, $\mathbf{P}$ is the Hodge projection onto the divergence free part, and where $u(v) = \mathbf{P}\left(\left(\mathcal{L}_v v^\flat\right)^\sharp\right)$. The flat and sharp symbols denote the index lowering and raising operators induced by the metric, i.e. the operators that convert vectors to one forms and vice versa.

With Krishnaprasad, Marsden and Murray I have been studying the geometry and dynamics of mechanical systems with nonholonomic constraints. Such constraints arise in many important mechanical systems including robotic systems. In many cases the equations of motion are of the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}^i}\right) - \frac{\partial L_c}{\partial r^i} = \left(\frac{\partial L}{\partial s^a}\right)\beta^a_{ij}\dot{r}^j \tag{1.3}$$

where L_c is the Lagrangian with the constraints substituted and the β's are curvature terms. Given a symmetry in the problem and defining the nonholonomic momentum map to be

$$J^c_\xi = \frac{\partial L}{\partial \dot{q}^i}(\xi^q_Q)^i , \tag{1.4}$$

we showed one gets generalized conservation laws of the form

$$\frac{dJ^c_\xi}{dt} = \frac{\partial L}{\partial \dot{q}^i}\left[\frac{d}{dt}(\xi^q)\right]^i_Q \tag{1.5}$$

which are useful for analyzing both the dynamics and control. In particular it enables one to relate dynamics in the variables directly controlled to the induced dynamics of the remaining variables. In local coordinates the momentum equation may be written in the form

$$\frac{d}{dt}J_b = \sum_{c=1}^{m}\sum_{l=1}^{n}\Gamma^c_{bl}J_c\dot{q}^l + \sum_{c=m+1}^{k}\sum_{i,l=1}^{n}\frac{\partial L}{\partial \dot{q}^i}\Gamma^c_{bl}\dot{q}^l e^i_c \tag{1.6}$$

which is a generalization of a parallel transport equation and reduces to one in special cases. I have also been studying the control coupling that arises in such equations with my student Scott Gray. In related work my student Dmitry Zenkov has been studying integrable nonholonomic systems and has shown that certain such systems exhibit periodic behavior in similar fashion to holonomic integrable Hamiltonian systems.

With McClamroch and Reyhanoglu I have been studying nonholonomic nonlinear control systems of the form

$$\dot{x} = f(x,z) + \sum_{i=1}^{m} g_i(x,z)u_i , \tag{1.7}$$

$$y = h(x,z) , \tag{1.8}$$

$$\dot{z} = S(y,z)\dot{y} . \tag{1.9}$$

We are able to derive controllability results for a general class of systems of this type, i.e. systems where the base (x,y) dynamics is input output decouplable. Such systems occur,

for example, when one takes into account actuator dynamics in a nonlinear control system and thus are useful for a detailed analysis of a mechanical system. Our results used some work of Coron and of Sussmann on dynamic extensions.

With Drakunov, I have been studying stabilization of nonholonomic control systems by nonsmooth feedback (such systems are never smoothly stabilizable). Using sliding mode type techniques we have been able to design a stabilizer for Brockett 's fundamental example (the Heisenberg system):

$$\begin{aligned} \dot{x} &= u \\ \dot{y} &= v \\ \dot{z} &= xv - yu. \end{aligned} \tag{1.10}$$

The control law is (up to a constant factor)

$$\begin{aligned} u &= -x + ysign(z) \\ v &= -y - xsign(z). \end{aligned} \tag{1.11}$$

We can show that with this feedback the condition for the system to be stabilized is

$$\frac{1}{2}[x^2(0) + y^2(0)] \geq |z(0)|. \tag{1.12}$$

This is a paraboloid in the phase space, but within the paraboloid we showed that one can use constant feedback to emerge from the parabloid, thus giving a globally stablizing controller. Simulation results for this system are very good. We have extended this result to more general classes of systems, in particular to all zero drift control systems which are linearly uncontrollable, but are nonlinearly controllable and are of nonholonomy degree one. This is a large and very important class of canonical systems first identified by Roger Brockett.

With Crouch and Ratiu I have been studying various aspects of optimal control both for nonholonomic systems and for more general nonlinear control systems. In particular we have analyzed the so called subRiemannian rigid body problem which corresponds to optimal control of a system on $SO(3)$ where one has only two controls. We were able to give a Lagrangian method for deriving the optimal controls and relate it to the Hamiltonian picture, and to the theory of integrable Hamiltonian systems. For a general class of systems with symmetries, where there is a smaller number of controls than the dimension of the state space, we showed one gets optimal control equations of the form $J\dot{V} = [Q, V]$, $Q = [JV, V]$. Here V represents the state variables and Q the costate variables. These are particularly amenable to explicit analysis. More recently, with Crouch I have extended these results to the analysis of the optimal control of a broad class of nonlinear control systems. Our key idea was to apply Lagrangian reduction techniques to constrained Lagrangians of the form

$$\Lambda(x, \dot{x}, \lambda) = L(x, \dot{x}) + \lambda^T \Phi(x, \dot{x}).$$

In particular we related the optimal control equations to the Lie-Poisson and Euler-Poincare equations of classical mechanics. This provides a singular extension of the recent Lagrangian

reduction analysis of Marsden and Scheurle. I also analyzed the optimal control of certain finite- and infinite-dimensional systems on adjoint orbits, showing how the optimal control equations were in double bracket form and extending work of Roger Brockett in this regard. We have also demonstrated that some of these optimal control problems are explicitly solvable. More recently we have analyzed higher order optimal control problems of this type.

With Alan Markworth and other workers at Batelle labs I have been examining the control of systems associated with anti-corrosion processes. This is both useful from a technological point of view and interesting from a theoretical point of view as the free system contains, in general, a strange attractor and exhibits chaotic behavior. We have been able to stabilize this system using classical control methods as well as more modern methods employing saddle-point structures. In particular, in the case of a two dimensional model, we were able to drive the system from a periodic attractor to a previously unstable saddle point. In this case the nonlinear equations describing the system take the form

$$\dot{\theta} = Y(1-\theta) - \theta e^{-\beta\theta} \tag{1.13}$$

$$\dot{Y} = p(1-\theta) - qY \tag{1.14}$$

Here we can control the paramter p via changing an anodic potential.

With Brockett, Flaschka and Ratiu I have been studying certain partial differential equations associated with the group of measure preserving diffeomorphisms of the annulus. This leads to systems which exhibit shocks, and the notion of measure-valued solutions, and to a number of interesting connections with fluid flows in two dimensions and ideas from control theory. This work extends our other work on integrable finite-dimensional dynamical systems such as the Toda lattice flow and their connections with gradient flows, steepest descent equations and various discrete algorithms, such as the QR algorithm and linear programming. The key in these finite-dimensional results was the existence of a polytope which made the link between the mechanical system and linear programming. This link was made via the geometry of the moment map. For our infinite-dimensional systems we proved the existence of a similar, but in this case, infinite-dimensional, polytope. The key equation in this setting takes the form of the gradient flow of the function $H(x(z,\theta)) = -\langle x(z,\theta), z\rangle$ on an orbit of of the group of diffeomorphims and is explicitly given by

$$x_t(z,\theta,t) = \{x(z,\theta,t), \{x(z,\theta,t), z\}\}.$$

This equation can in fact be shown to solve an infinite-dimensinal linear programming problem over a suitably defined polytope. The equilibria of the equation lie at the vertices of this polytope and corresond to equimeasureable rearrangements of the function $x(z,t)$.

In a special case these equations become the pair of hyperbolic equations in two dependent variables

$$\frac{\partial v}{\partial t} = v\frac{\partial u}{\partial z}, \qquad \frac{\partial u}{\partial t} = 2\frac{\partial v^2}{\partial z}.$$

These special equations are very interesting, being a direct generalization of the finite Toda lattice equations. They are also gradient-like, minimizing a suitable function, and are integrable. Further, like finite Toda, they possess a sorting property.

With Kappeler and coworkers I have studied related integrable equations, in particular the Korteweg de Vries equations, the periodic Toda lattice and the defocusing nonlinear Schroedinger equation. We showed that the phase spaces for these systems are diffeomorphic to that for decoupled harmonic oscillators, thus giving a very complete picture of their dynamics.

Recently, with a postdoc I have been examining factoization solutions to such problems.

Professional personel associated with the research effort:

This grant has been supporting the graduate work of two graduate students at Ohio State: Scott Gray and Dmitry Zenkov (summer support and some some term support) and the collaborative research work of Sergey Drakunov, a postdoctoral research engineer at Ohio State.

Interaction

Meetings etc.:

"Nonholonomic Control Systems," Fields Institute Workshop on Falling Cats, The Fields Institute, Waterloo, Ontario, March, 1992.

"Convexity and Integrability," Fields Institute Workshop on Gradient and Hamiltonian Flows, Algorithms and Control, The Fields Institute, Waterloo, Ontario, April, 1992.

"The Geometry of Nonholonomic Control Systems," NSF Workshop on Nonlinear Control, St. Louis, May, 1992.

"On the Dynamics and Control of Nonholonomic Systems on Riemannian Manifolds," IFAC Nonlinear Control Systems Symposium, Bordeaux, France, June, 1992.

"The Geometry of Nonholonomic Systems on Riemannian Manifolds," SIAM Conference on Control, Minneapolis, September, 1992.

"Control of Systems with Homoclinic and Heteroclinic Orbits," SIAM Conference on Dynamical Systems, Salt Lake City, October, 1992.

"Integrable Differential Equations and Convexity," The 876th Meeting of the American Mathematical Society, Dayton, October, 1992.

"Controllability of Nonholonomic Systems on Riemannian Manifolds," The 31st IEEE Conference on Decision and Control, Tucson, December, 1992.

" On the Geometry of Saddle Point Algorithms," The 31st IEEE Conference on Decision and Control, Tucson, December, 1992.

"The Dynamics of Gradient and Hamiltonian Flows and Convexity," Dynamics Days, Tempe, Arizona, January, 1993.

"The Dynamics of Generalized Rigid Bodies," Workshop on Robotics, The Institute for Mathematics and its Applications, Minnesota, January, 1993.

"The Dynamics of the Toda Lattice in Finite and Infinite Dimensions," Workshop on Geometric Mechanics, Rio de Janeiro, March, 1993.

"Gradient and Hamiltonian Flows in Infinite Dimensions," Geometric, Analytic, and Computational Aspects of Mechanics, Oberwolfach, July, 1993.

"Hamiltonian Flows in Networks and Control," The International Symposium on the Mathematical Theory of Networks and Control 93, Regensburg, August, 1993.

"Optimal Control and Vakonomic Mechanics," The Joint American Mathematical Society – Canadian Mathematical Society Summer Meeting, Vancouver, August, 1993.

"The Geometry of Nonholonomic and Vakonomic Control Systems," The Workshop on Mechanics, Holonomy and Control, San Antonio, December, 1993.

"Convexity, Group Theory and the Dispersionless Toda Flow," Cornelius Lanczos International Centenary Conference, Raleigh, December, 1993.

"Symmetry, Constraints and Dissipation," MSRI Workshop on Exterior Differential Systems, Submanifolds and Control Theory, MSRI, Berkeley, March, 1994.

"An Infinite-dimensional Optimization Problem," IMACS World Congress, Atlanta, July 1994.

"Examples in Nonholonomic Mechanics," Workshop on Geometric Mechanics and Nonholonomic Systems, Berkeley, August, 1994.

" Stabilization of a Nonholonomic System via Sliding Modes," The 33rd IEEE Conference on Decision and Control, Orlando, December, 1994.

"Dynamics of the Toda Lattice and Convexity", hour talk given at the Midwest Dynamical Systems Meeting, Minneapolis, March, 1995.

"Lagrangian and Hamiltonian Formulations of Constrained Variational Problems," The Meeting on Geometric Mechanics, Dynamical Systems and Control, Fayetteville, April, 1995.

"Optimal Control and the Lagrange Problem," Third SIAM Conference on Control and its Applications, St. Louis, April, 1995.

"Constrained Variational Prinicples and Optimal Control," The Third International Congress on Industrial and Applied Mathematics, Hamburg, Germany, July, 1995.

"The Geometry of Nonholonomic Mechanical Systems with Symmetry," The Third International Congress on Industrial and Applied Mathematics, Hamburg, Germany, July, 1995.

"Multiple brackets and Optimal Control," ARO-NASA Workshop on Exterior Differential Systems and Hybrid Control, Bozeman, Montana, July, 1995.

"Discrete Computation and Smooth Hamiltonian and Gradient Flows," Workshop on New Connections between Mathematics and Computer Science, The Newton Institute, Cambridge, November, 1995.

"On the geometry of optimal control and geodesic flows," The 34th IEEE Conference on Decision and Control, New Orleans, December, 1995.

"Integrable geodesic flows on homogeneous spaces," Joint Mathematics Meeting, Orlando, January, 1996.

"Stabilization of nonlinear control systems," The AFSOR Contractors/Grantees Meeting, Pasadena, March, 1996.

Consultations:

Work and consulting with Alan Markworth, Battelle Labs, Columbus, OH, several meetings over a two year period. Visit to Ford Scientific Research Lab.

Papers

Motion planning for nonholonomic dynamic systems, in *Nonholonomic Motion Planning* (Z. Li and J. F. Canny eds.), Kluwer, 1993, 210-234 (with N. H. McClamroch and M. Reyhanoglu).

On the dynamics and control of nonholonomic systems on Riemannian manifolds, *Proceedings of the IFAC Nonlinear Control Symposium, 1992* (with P. E. Crouch).

The Whitham equation and shocks in the Toda lattice, in *Singular Limits of Dispersive Waves* (N. Ercolani, I. Gabitov, C. Levermore and D. Serre eds.), Plenum Press, 1994, 1-19 (with Y. Kodama).

Controllability of nonholonomic systems on Riemannian manifolds, *The Proceedings of the 31st IEEE Conference on Decision and Control*, IEEE (1992), 1594-1596 (with P. E. Crouch).

On the geometry of saddle point algorithms, *The Proceedings of the 31st IEEE Conference on Decision and Control*, IEEE(1992), 1482-1487 (with R. W. Brockett and T. S. Ratiu).

Nonholonomic control systems on Riemannian manifolds, *The SIAM Journal on Control and Optimization* **37** No. 1, (1995), 126-148 (with P. E. Crouch).

Nonholonomic and vakonomic control systems on Riemannian manifolds, *Fields Institute Communications* 1, (1993) 25-52 (with P. E. Crouch).

Dissipation induced instabilities, *Annales de l'Institut Henri Poincaré, Analyse Non Linéaire* 11, 37-90 (1994) (with P. S. Krishnaprasad, J. E. Marsden, and T. S. Ratiu).

A Schur-Horn-Kostant convexity theorem for the diffeomorphism group of the annulus, *Inventiones Mathematicae* 113, 511-529 (1993) (with H. Flaschka and T. S. Ratiu).

Sub-Riemannian optimal control problems and the sub-Riemannian rigid body, *Fields Institute Communications* 3, (1994), 35-48 (with P. E. Crouch and T. S. Ratiu).

The Toda PDE and the Geometry of the Diffeomorphism Group of the Annulus, *Fields Institute Communications* **7** (1996), 57-92, (with H. Flaschka and T. S. Ratiu).

Control and stabilization of a general class of nonholonomic dynamic systems, *The Proceedings of the 32nd IEEE Conference on Decision and Control*, IEEE (1993) (with N. H. McClamroch and M. Reyhanoglu).

La structure symplectique de l'espace de phase de l'equation de Korteweg de Vries, *C. R. Acad. Sci* **317**, 1019-1022 (1993) (with D. Bättig, J. C. Guillot, and T. Kappeler).

On the symplectic structure of the phase space for the periodic KdV, Toda and defocusing NLS, *Duke Mathematical Journal* **79** (1996), 549-604. (with D. Bättig, J. C. Guillot, and T. Kappeler).

The Euler-Poincaré equations and double bracket dissipation, *Communications in Mathematical Physics* **175**, 1-42 (1996)(with P. S. Krishnaprasad, J. E. Marsden, and T. S. Ratiu).

Control of nonholonomic systems with extended base space dynamics, *The International Journal on Robust and Nonlinear Control* **5** (1996), 325-330(with N. H. McClamroch and M. Reyhanoglu).

Control and optimal control of a class of infinite-dimensional systems, in *Proceedings of the IMACS World Congress on Computational and Applied Mathematics* (ed. W. F. Ames), 51-55, Georgia Institute of Technology, 1994.

Stabilization of a nonholonomic system via sliding modes, in *The Proceedings of the 33rd IEEE Conference on Decision and Control*, 2961-2964, IEEE (1994) (with S. Drakunov).

Linear-feedback-induced-control of relaxation oscillations in a metal-passivation model, to appear (with Alan Markworth).

Reduction of Euler Lagrange problems for constrained variational problems and relation with optimal control problems, in *The Proceedings of the 33rd IEEE Conference on Decision and Control*, 2584-2590, IEEE (1994) (with P. Crouch).

Nonholonomic mechanical systems with symmetry, to appear in the *Archive for Rational Mechanics and Analysis* (with P. S. Krishnaprasad, J. E. Marsden and R. Murray).

Tracking in nonholonomic dynamic systems via sliding modes, in *The Proceedings of the 35th IEEE Conference on Decision and Control*, 2103-2106, IEEE (1995) (with S. Drakunov).

Optimal control on adjoint orbits and symmetric spaces, in *The Proceedings of the 35th IEEE Conference on Decision and Control*, 3283-3288, IEEE (1995) (with P. Crouch).

Optimal control and geodesic flows, to appear in *Systems and Control Letters* (with P. Crouch).

Symmetries and integrability of nonholonomic systems, to appear (with P. Crouch).

Stabilization and tracking in the nonholonomic integrator via sliding modes, to appear (with S. Drakunov).

Another view of nonholonomic mechanical control systems, in *The Proceedings of the 34th IEEE Conference on Decision and Control*, 1066-1071, IEEE (1995) (with P. Crouch).

Double bracket equations and geodesic flows on symmetric spaces, to appear (with R. Brockett and P. Crouch).

Hamiltonian and gradient flows, algorithms and control, Fields Institute Communications, AMS, 1994 (editor of book).
Student Publication: D. Zenkov, On the geometry of the Routh problem, *Journal of Nonlinear Science* appear.